4th Grade Common Core State Standards

Flip Book

This document is intended to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. Resources used: CCSS, Arizona DOE, Ohio DOE and North Carolina DOE. This "Flip Book" is intended to help teachers understand what each standard means in terms of what students must know and be able to do. It provides only a *sample* of instructional strategies and examples. The goal of every teacher should be to guide students in understanding & making sense of mathematics.

Construction directions:

Print on cardstock. Cut the tabs on each page starting with page 2. Cut the bottom off of this top cover to reveal the tabs for the subsequent pages. Staple or bind the top of all pages to complete your flip book.

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Mathematical Practice Standards (MP) summary of each standard

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students in fourth grade know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2. **Reason abstractly and quantitatively.**
   Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3. **Construct viable arguments and critique the reasoning of others.**
   In fourth grade, mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4. **Model with mathematics.**
   Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.
5. **Use appropriate tools strategically.**
   Mathematically proficient fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units
6. **Attend to precision.**
   As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

7. **Look for and make use of structure. (Deductive Reasoning)**
   In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8. **Look for and express regularity in repeated reasoning. (Inductive Reasoning)**
   Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _______ related to ________? What is the relationship between ______and ______? What does_______mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a ______ show us that _____may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?

Critical Areas for Mathematics in 4th Grade

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $^{15}/_{9} = {}^{5}/_{3}$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Domain: **Operations and Algebraic Thinking**

Cluster: **Use the four operations with whole numbers to solve problems.**

Standard: **4.OA.1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.

Connections (4.OA. 1-3):

This cluster is connected to the Fourth Grade Critical Area of Focus #1. **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.**

Represent and solve problems involving multiplication and division (Grade 3 OA 3).

Solve problems involving the four operations, and identify and explain patterns in arithmetic (Grade 3 OA 8).

Explanations and Examples

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given many opportunities to write and identify equations and statements for multiplicative comparisons. It is essential that students are provided many opportunities to solve contextual problems.

Example:

Multiplicative Comparison

5 x 8 = 40.

Sally is five years old. Her mom is eight times older. How old is Sally's Mom?

5 x 5 = 25

Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

Instructional strategies for OA.1-3 next page

Instructional Strategies (4.OA.1-3)
Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 page 74. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2).

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Examples of multistep word problems can be accessed from the released questions on the NAEP (National Assessment of Educational Progress) Assessment at http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx.

For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, "Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?"

Instructional Resources/Tools
Table 2 page 74
National Assessment of Educational Progress (NAEP) Assessments - http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx.

Continued next page

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Use the four operations with whole numbers to solve problems.

Standard: **4.OA.2.** Multiply or divide to solve word problems involving multiplicative comparison*,* e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison*.* (see Table 2)

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:

See 4.OA.1

Explanations and Examples:

This standard calls for students to translate comparative situations into equations with an unknown and solve.
Students need many opportunities to solve contextual problems. Refer to Table 2 page 74 for more examples (table included at the end of this document for your convenience).

Examples:

Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? (3 x 6 = p).

Group Size Unknown: A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? (18 ÷ p = 3 or 3 x p = 18).
Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? (18 ÷ 6 = p or 6 x p = 18).

When distinguishing multiplicative comparison from additive comparison, students should note that:

- *additive comparisons* focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, "How many more?"
- *multiplicative comparisons* focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is "How many times as much?" or "How many times as many?"

More examples next page

Students need many opportunities to solve contextual problems. Table 2 includes the following multiplication problem:

> "A blue hat costs $6. A red hat costs 3 times as much as the blue hat.
> How much does the red hat cost?"

In solving this problem, the student should identify $6 as the quantity that is being multiplied by 3. The student should write the problem using a symbol to represent the unknown.

($6 x 3 = ☐)

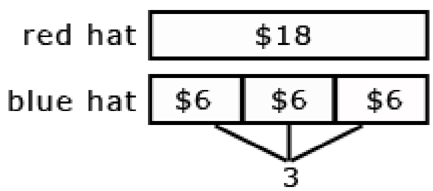
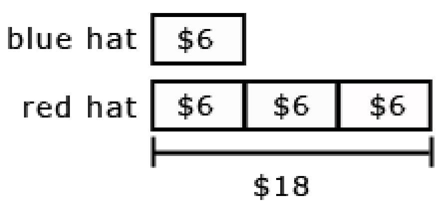


Table 2 includes the following division problem:

> A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?

In solving this problem, the student should identify $18 as the quantity being divided into shares of $6.

The student should write the problem using a symbol to represent the unknown.
($18 ÷ $6 = ☐)



Domain: **Operations and Algebraic Thinking**

Cluster: **Use the four operations with whole numbers to solve problems.**

Standard: **4.OA.3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Standards for Mathematical Practice (MP) to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use

Connections:

See 4.OA.1

Explanations and Examples:

The focus in this standard is to have students use and discuss various strategies. It refers to **estimation** strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? Some typical estimation strategies for this problem:

Student 1
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2
I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When
I add that hundred to the 4 hundreds that I already had, I end up with 500.

Student 3
I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Examples continued next page

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.

Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300 - 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. ways to address remainders:
- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:
Problem A: 7
Problem B: 7 r 2
Problem C: 8
Problem D: 7 or 8
Problem E: 7 2/6

possible solutions:
Problem **A: 7.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*
Problem **B: 7 r 2.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*
Problem **C: 8.** Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*
Problem **D: 7 or 8.** Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*
Problem **E: 7 2/6**
Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* **7 2/6**

Additional examples next page

Example:
There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over.

Example:

Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?

3 x $12 + $15 = *a*

In division problems, the remainder is the whole number left over when as large a multiple of the divisor as possible has been subtracted.

Example:

Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now?
(7 bags with 4 leftover)

Kim has 28 cookies. She wants to share them equally between herself and 3 friends. How many cookies will each person get?
(7 cookies each) 28 ÷ 4 = *a*

There are 29 students in one class and 28 students in another class going on a field trip. Each car can hold 5 students. How many cars are needed to get all the students to the field trip?
(12 cars, one possible explanation is 11 cars holding 5 students and the 12^{th} holding the remaining 2 students) 29 + 28 = 11 x 5 + 2

Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to:
- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

Students need many opportunities solving multistep story problems using all four operations.
An interactive whiteboard, document camera, drawings, words, numbers, and/or objects may be used to help solve story problems.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite**

Standard: **4.OA.4**. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.7. Look for and make use of structure.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #1, Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.

Understand properties of multiplication and the relationship between multiplication and division (Grade 3 OA 5 – 6).

Geometric measurement: understand concepts of area and relate area to multiplication and to addition (Grade 3 MD 7a).

The concepts of prime, factor and multiple are important in the study of relationships found among the natural numbers. Compute fluently with multi-digit numbers and find common factors and multiples (Grade 6 NS 4).

Explanations and Examples:

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.

Students investigate whether numbers are prime or composite by

• building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
• finding factors of the number

Continued next page

Students should understand the process of finding factor pairs so they can do this for any number 1 – 100.

Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8,12, 24

Multiples: 1, 2, 3, 4, 5…24
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
3, 6, 9, 12, 15, 18, 21, 24
4, 8, 12, 16, 20, 24
8, 16, 24
12, 24
24

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints
include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

<u>Instructional Strategies:</u>

Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

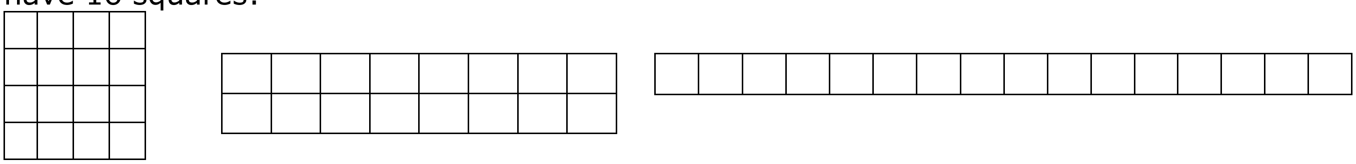
Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12,… are composite. Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.

Instructional strategies continued next page

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number. They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should **not** be provided, but determined after many strategies have been used in finding all possible factors of a number.
Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers.
Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:



The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example. 5 x 8 is 40; the table below shows the multiples of each factor.

5	10	15	20	25	30	25	40	45
8	16	24	32	40	48	56	64	72

Ask students what they notice about the number 40 in each set of multiples; 40 is the 8th multiple of 5, and the 5th multiple of 8.

Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.

Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

Resources:
nctm.org (Illuminations): *The Factor Game* engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.

nctm.org (Illuminations): *The Product Game* – Classifying Numbers. Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the Product Game.
nctm.org(Illuminations) *The Product Game*--In the Product Game, students start with factors and multiply to find the product. In The Factor Game, students start with a number and find its factors.
nctm.org (Illuminations): *Multiplication: It's in the Cards* – More Patterns with Products.

Common Misconceptions:
When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.
Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

Domain: **Operations and Algebraic Thinking (OA)**

Cluster: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **pattern (number or shape), pattern rule**

Standard: **4.OA.5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision
MP.7. Look for and make use of structure.
MP. 8. Look for and express regularity in repeated reasoning.

Connections:

This cluster goes beyond the Fourth Grade Critical Areas of Focus to address **Analyzing patterns**.

Solve problems involving the four operations, and identify and explain patterns in arithmetic. (3.OA.4.9).

Explanations and Examples:

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Examples:

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28, …	Start with 3, add 5	The numbers alternately end with a 3 or 8
5, 10, 15, 20 ….	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:

Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, etc.)

Continued next page

In this standard, students **describe** features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5

Day	Operation	Beans
0	3x0+4	4
1	3x1+4	7
2	3x2+4	10
3	3x3+4	13
4	3x4+4	16
5	3x5+4	19

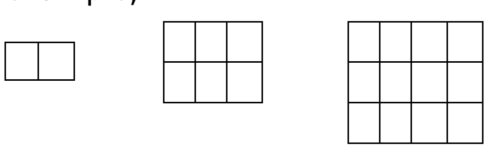
Instructional Strategies:
In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4" starting with the number 1, the pattern 1, 5, 9, 13, 17, … is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example,



Students may state that the rule is to multiply the previous number of squares by 3.

Resources continued next page

Instructional Resources/Tools:
From PBS Teachers: Snake Patterns –s-s-s: Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.
http://www.pbs.org/teachers/mathline/lessonplans/atmp/snake/snake_procedure.shtm

nctm.org (Illuminations) *Patterns that Grow – Growing Patterns*. Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal's triangle.

nctm.org (Illuminations): *Patterns that Grow – Exploring Other Number Patterns*. Students analyze numeric patterns, including Fibonacci numbers. They also describe numeric patterns and then record them in table form.

nctm.org (Illuminations): *Patterns that Grow – Looking Back and Moving Forward*. In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.

Extended Common Core State Standards
Mathematics

The *Alternate Achievement Standards for Students With the Most Significant Cognitive Disabilities.* Materials should show a clear link to the core standards for the grade in which the student is enrolled, although the grade-level content may be reduced in complexity or modified to reflect pre-requisite skills." Throughout the Standards descriptors such as, describe, count, identify, etc, should be interpreted to mean that the students will be taught and tested according to their mode of communication. (North Carolina DOE)

Fourth Grade Mathematics Operations and Algebraic Thinking (4.OA)				
	Common Core State Standards	**Essence**		**Extended Common Core**
	Use the four operations with whole numbers to solve problems	**Use operations to solve problems**		**Use the two operations with whole numbers to solve problems (up to 50)**
Cluster	1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.		Cluster	1. Solve addition and subtraction problems when change is unknown (i.e. 8 + = 10, 6 – = 3). 2. Use part-part-whole problem, to combine two parts into one whole when whole is unknown. (See Table 1 in Core pg. 88)
	Gain familiarity with factors and multiples	**Build understanding of multiplication and division**		**Understand relationship between and division**
	4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.			3. Illustrate multiplication and division by making equal sized groups using models. 4. Understand that even numbers are sets that can be shared equally between 2 people and odd sets cannot. 5. Use the symbolic representation of multiplication and division to write a number sentence.
	Generate and analyze patterns	**Analyze Patterns**		**Analyze Patterns**
	5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.			6. Use repeating shape patterns to make predictions and extend simple repeating patterns. 7. Understand the concept of counting by 2's.

EXTENDED 4.OA.1-5

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round**

Standard: **4.NBT.1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right*. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections: **(4.NBT.1-3)**

This cluster is connected to the Fourth Grade Critical Area of Focus #1, Developing an understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends.

A strong foundation in whole-number place value and rounding is critical for the expansion to decimal place value and decimal rounding.

Understand place value (Grade 2 NBT 1 – 4).

Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 3 NBT 1).

Explanations and Examples:

This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

Example:

How is the 2 in the number 582 similar to and different from the 2 in the number 528?

Students should be familiar with and use place value as they work with numbers. Some activities that will help students develop understanding of this standard are:

- Investigate the product of 10 and any number, then justify why the number now has a 0 at the end. (7 x 10 = 70 because 70 represents 7 tens and no ones, 10 x 35 = 350 because the 3 in 350 represents 3 hundreds, which is 10 times as much as 3 tens, and the 5 represents 5 tens, which is 10 times as much as 5 ones.) While students can easily see the pattern of adding a 0 at the end of a number when multiplying by 10, they need to be able to justify why this works.
- Investigate the pattern, 6, 60, 600, 6,000, 60,000, 600,000 by dividing each number by the previous number.

Instructional Strategies next page

Instructional Strategies: (4.NBT.1-3)
Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits, e.g., compare 453, 698 and 215; numbers that have the same number in the leading digit position, e.g., compare 45, 495 and 41,223; and numbers that have different numbers of digits and different leading digits, e.g., compare 312, 95, 5245 and 10,002.

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit, e.g., round 23,960 to the nearest hundred. This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of; "Is 48 closer to 40 or 50?" Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit.

Instructional Resources/Tools
Place value flip charts
Number cards

Common Misconceptions: (4.NBT.1-3)
There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.

Domain: **Number and Operation in Base Ten**

Cluster: 1 Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round**

Standard: **4.NBT.2.** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections:

See 4.NBT.1

Explanations and Examples:

This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.

Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

The expanded form of 275 is 200 + 70 + 5. Students use place value to compare numbers. For example, in comparing 34,570 and 34,192, a student might say, both numbers have the same value of 10,000s and the same value of 1000s however, the value in the 100s place is different so that is where I would compare the two numbers.

Domain: **Number and Operations in Base Ten (NBT)**

Cluster: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, <, >, =, comparisons/compare, round**

Standard: **4.NBT.3.** Use place value understanding to round multi-digit whole numbers to any place.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.6. Attend to precision.

Connections:

See 4.NBT.1

Explanations and Examples:

This standard refers to place value understanding, which extends **beyond** an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds.
So we still need 250 bottles.

Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40.
40+20=60. 300- 60 = 240, so we need about 240 more bottles.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel? Some typical estimation strategies for this problem:

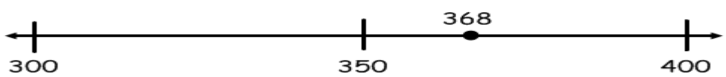
Student 1
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2
I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.

Student 3
I rounded 267 to
300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

Examples continued next page

Example:
Round 368 to the nearest hundred.
This will either be 300 or 400, since those are the two hundreds before and after 368.
Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400.
Since 368 is closer to 400, this number should be rounded to 400



When students are asked to round large numbers, they first need to identify which digit is in the appropriate place.

Example or reasoning: Round 76,398 to the nearest 1000.

- Step 1: Since I need to round to the nearest 1000, then the answer is either 76,000 or 77,000.
- Step 2: I know that the halfway point between these two numbers is 76,500.
- Step 3: I see that 76,398 is between 76,000 and 76,500.
- Step 4: Therefore, the rounded number would be 76,000.

<u>Common Misconceptions:</u>
See 4.NBT.1

Domain: **Number and Operation in Base Ten**

Cluster: **Use place value understanding and properties of operations to perform multi-digit arithmetic.** Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round**

Standard: **4.NBT.4.** Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(4.NBT.4-6)**

This Cluster is connected to the Fourth Grade Critical Areas of Focus #1 , Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends, and go beyond to address Adding and subtracting multi-digit whole numbers.

Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 2 – 3)

Use the four operations with whole numbers to solve problems (Grade 4 OA 2 – 3).
Generalize place value understanding for multi-digit whole numbers (Grade 4 NBT 1 – 2).

Explanations and Examples:

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to fluency, which means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property, decomposing and recomposing numbers, etc.). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

$$\begin{array}{r} 3892 \\ +\underline{1567} \end{array}$$

See student explanation for this problem on next page:

Student explanation for this problem:

1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the10 tens as one more hundred.(notates with a 1 above the hundreds column)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

3546 <u>- 928</u>

Student explanation for this problem:

1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)
2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)
3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).
6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

<u>Instructional Strategies</u>: (4.NBT.4-6)
A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about, rather than merely follow a sequence of directions, rules or procedures that they don't understand.

It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Continued next page

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers, provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

Common Misconceptions: (4.NBT.4-6)

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

Specific strategies or students having difficulty with lining up similar place values in numbers as they are adding and subtracting, it is sometimes helpful to have them write their calculations on grid paper. This assists the student with lining up the numbers more accurately.

If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the 'shorthand' of 'carrying,' students could add by place value, moving left to right placing the answers down below the 'equals' line. For example:

```
  249        (start with 200 + 300 to get the 500, then 40 + 70 to get 110, and 9 + 2 for 11)
 +372
  500
  110
   11
 ----
  621
```

Domain: **Number and Operation in Base Ten**

Cluster: **Use place value understanding and properties of operations to perform multi-digit arithmetic.** Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, <, >, =, comparisons/compare, round**

Standard: **4.NBT.5**. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:
See 4.NBT.4-6

Explanations and Examples:

Students who develop flexibility in breaking numbers apart (decomposing numbers) have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication and understanding why it works, is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

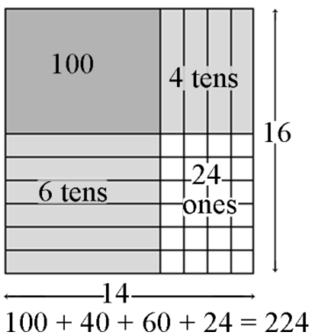
Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the baker?

Student 2	Student 3	Student 1
25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 50 x 6 = 300	25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300

Additional examples next page

Use of place value and the distributive property are applied in the scaffolded examples below.

- To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 X 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.
- The area model shows the partial products.
  14 x 16 = 224



Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

```
  25
X 24
 400 (20 x 20)
 100 (20 x 5)
  80 (4 x 20)
  20 (4 x 5)
 600
```

- 25

```
  25
 x24
 500 (20 x 25)
 100 (4 x 25)
 600
```

- **Matrix Model:** This model should be introduced after students have facility with the strategies shown above.

	20	5	
20	400	100	500
4	80	20	100
	480 + 120		600

Example:
What would an array area model of 74 X 38 look like?

	70	4
30	70 X 30 = 2,100	4x30=120
8	70 X 8 = 560	4x8=32

2,000=560+1,200+32=2,812

Domain: **Number and Operation in Base Ten**

Cluster: **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, <, >, =, comparisons/compare, round**

Standard: **4.NBT.6**. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:

See 4.NBT.4

Explanations and Examples:

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Examples:

A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks**: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
- **Using Multiplication:** 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

This standard calls for students to explore division through various strategies.

Student 1	**Student 2**	**Student 3**
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74 592 −400 | 50 192 −160 | 20 32 −32 | 4 0	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

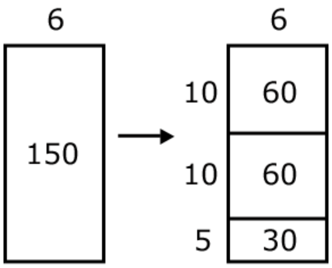
Continued next page

Example:

- **Using an Open Array or Area Model**

  After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5^{th} grade.

  150 ÷ 6



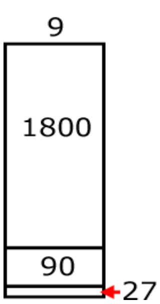
Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:
   a. 150   150 ÷ 6 = 10 + 10 + 5 = 25
   - 60 (6 x 10)
   90
   - 60 (6 x 10)
   30
   - 30 (6 x 5)
   0

   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

Example 2:

1917 ÷ 9



A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines.

1917 ÷ 9 = 213

EXTENDED MATHEMATICS
Fourth Grade

4th Grade Mathematics Number and Operations in Base Ten (4.NBT)				
	Common Core State Standards	**Essence**		**Extended Common Core**
	Generalize place value understanding for multi-digit whole numbers	**Place value understanding whole numbers**		**Generalize place value understanding for multi-digit whole numbers**
Cluster	1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 3. Use place value understanding to round multi-digit whole numbers to any place.		Cluster	1. Illustrate whole numbers to 50 by composing and decomposing numbers. 2. Use a number line or hundreds chart to compare numbers greater than, less than or equal to.
Cluster	**Use place value understanding and properties of operations to perform multi-digit arithmetic**	**Place value and properties of operations**	Cluster	**Use place value understanding and properties of operations to perform multi-digit arithmetic**
	4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. 5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.			3. Illustrate multiplication and division by making 2 equal sized groups up to 10.

Domain: **Number and Operation – Fractions**

Cluster: **Extend understanding of fraction equivalence and ordering.**
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.

Standard: **4.NF.1.** Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(4.NF.1-2)**
This cluster is connected to the Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Develop understanding of fractions as numbers (Grade 3 NF 3).

Explanations and Examples:
This standard refers to visual fraction models. This includes area models, linear models (number lines) or it could be a collection/set models.

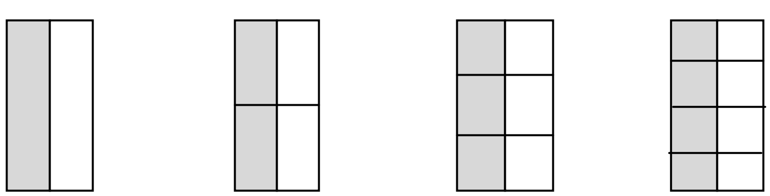
This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

Students can use visual models or applets to generate equivalent fractions.

Example:
All the area models show 1/2. The second model shows 2/4 but also shows that 1/2 and 2/4 are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved.

Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.

1/2 x 2/2 = 2/4.

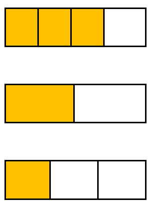


$\frac{1}{2}$ $\quad$ $\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$ $\quad$ $\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$ $\quad$ $\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$

Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80
Instructional Strategies next page

Instructional Strategies: (4.NF.1-2)

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions.
Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators.
Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The models should be represented in drawings. Students should also use **benchmark fractions** such as 1/2 to compare two fractions and explain their reasoning. The result of the comparisons should be recorded using >, < and = symbols.



3/4

1/2

1/3

Common Misconceptions: (4.NF.1-2)

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing 1/2 to sixths. They would multiply the denominator by 3 to get 1/6, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.
It's important that students use a fraction in the form of **one** such as 3/3 so that the numerator and denominator do not contain the original numerator or denominator.

Domain: **Number and Operation – Fractions**[1]

Cluster: **Extend understanding of fraction equivalence and ordering.**
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions.

Standard: **4.NF.2.** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:

See 4.NF.1

Explanations and Examples:

This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. **Students' experiences should focus on visual fraction models rather than algorithms**. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare and use reasoning skills based on fraction benchmarks. Students must also recognize that they must consider the size of the whole when comparing fractions (ie,1/2 and 1/8 of two medium pizzas is very different from1/2 of one medium and 1/8 of one large). Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Example:

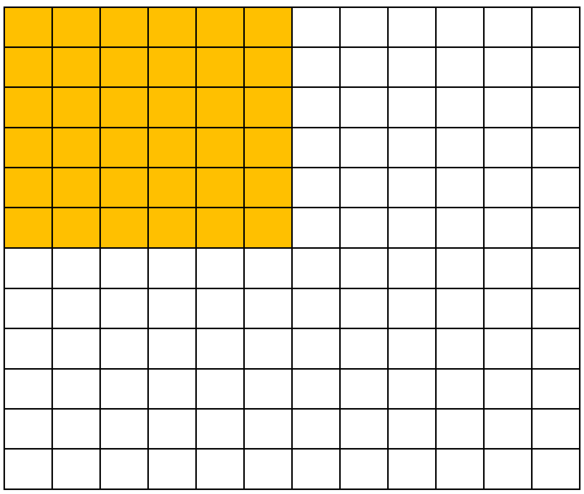
Use patterns blocks.

1. If a red trapezoid is one whole, which block shows 1/3 ?
2. If the blue rhombus is 1/3, which block shows one whole?
3. If the red trapezoid is one whole, which block shows 2/3?

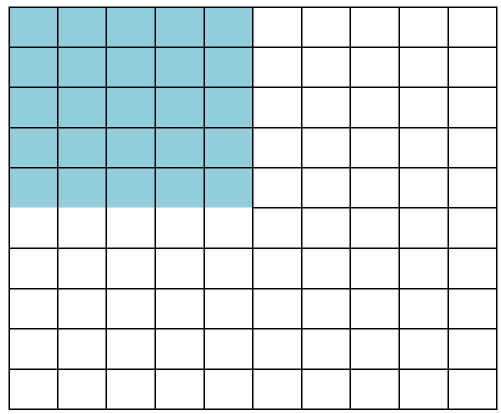
Examples continued next page

Melisa used a 12 x 12 grid to represent 1 and Nancy used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Melisa shade? How many grid squares did Nancy shade? Why did they need to shade different numbers of grid squares?

Possible solution: Melisa shaded 36 grid squares; Nancy shaded 25 grid squares. The total number of little squares is different in the two grids, so 1/3 *of each total number is different.*



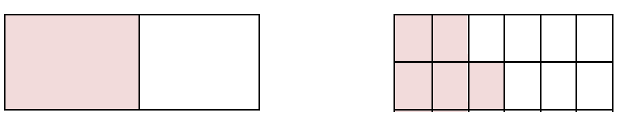
Melisa's grid



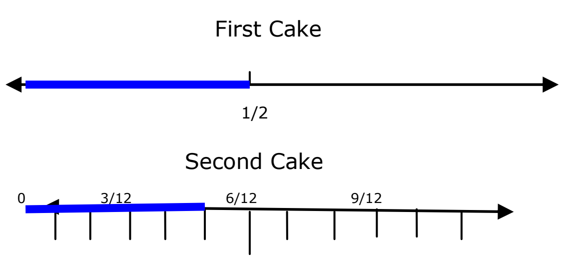
Nancy's grid

Example:
There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left?

Student 1
Area model:
The first cake has more left over. The second cake has 5/12 left which is smaller than ½.



Student 2
Linear/Number Line model:



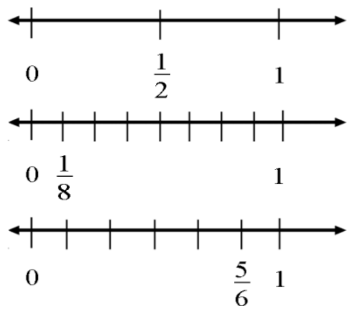
Student 3 explanation using Benchmark Fractions next page

Student 3:
I know that 6/12 equals 1/2. Therefore, the second cake which has 7/12 left is greater than 1/2.
Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =.

It is important that students explain the relationship between the numerator and the denominator, using Benchmark Fractions. See examples below:

Fractions may be compared using $\frac{1}{2}$ as a benchmark.



Possible student thinking by using benchmarks:

- $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

Possible student thinking by creating common denominators:

- $\frac{5}{6} > \frac{1}{2}$ because $\frac{3}{6} = \frac{1}{2}$ and $\frac{5}{6} > \frac{3}{6}$

Fractions with common denominators may be compared using the numerators as a guide.

- $\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$

Fractions with common numerators may be compared and ordered using the denominators as a guide.

$\frac{3}{10} < \frac{3}{8} < \frac{3}{4}$

Domain: **Number and Operations—Fractions (NF)**

Cluster: **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Standard: **4.NF.3.** Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
   Examples: $3/8=1/8+1/8+1/8$; $3/8=1/8+2/8$; $2\ 1/8=1 + 1+1/8=8/8+8/8 +1/8$.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem

Standards for Mathematical Practice (MP) to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections: **(4.NF.3-4)**

This cluster is connected to the Fourth Grade Critical Area of Focus #2 , **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Represent and interpret data (Grade 4 MD 4).

Explanations and Examples:

A fraction with a numerator of one is called a **unit fraction.** When students investigate fractions other than unit fractions, such as 2/3, they should be able to decompose the non-unit fraction into a combination of several unit fractions.

Example: 2/3 = 1/3 + 1/3

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example:

- 1 ¼ - ¾ = ☐

  4/4 + ¼ = 5/4

  5/4 – ¾ = 2/4 or ½

Continued next page

Example of word problem:

- Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?

  Solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:

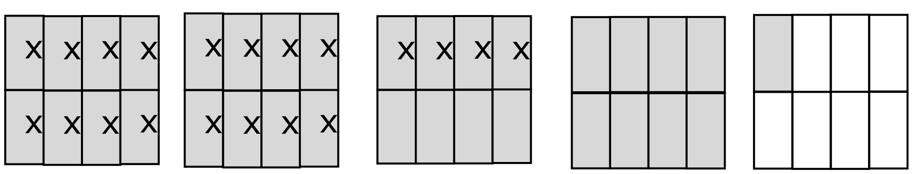
- Susan and Avery need 8 3/8 feet of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Avery has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Avery has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Avery has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot.

Example:

- Timothy has 4 1/8 pizzas left over from his soccer party. After giving some pizza to his friend, he has 2 4/8 of a pizza left. How much pizza did Timothy give to his friend?

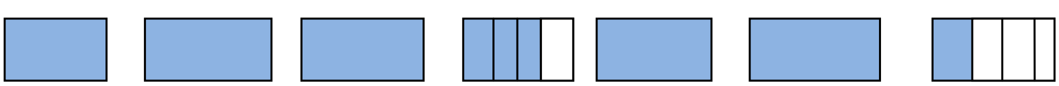
Solution: Timothy had 4 1/8 pizzas to start. This is 33/8 of a pizza. The x's show the pizza he has left which is 2 4/8 pizzas or 20/8 pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is 13/8 or 1 5/8 pizzas.



Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions. Keep in mind **Concrete-Representation-Abstract (CRA)** approach to teaching fractions. Students need to be able to "show" their thinking using concrete and/or representations BEFORE they move to abstract thinking.

Example:

While solving the problem 3 ¾ = 2 ¼' students could do the following:

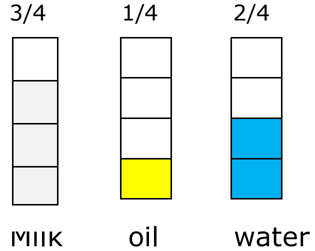


Student 1	Student 2	Student 3
3 + 2 = 5 and ¾ + ¼ = 1 so 5+ 1 = 6	3 ¾ + 2 = 5 ¾ + ¼ = 6	3 ¾ = 1 5/4 and 2 ¼ = 9/4 so 15/4 + 9/4 = 24/4 = 6

Additional examples next page

Example:

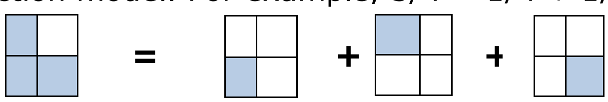
A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake? Use an area model to solve.



6/4 or 1 whole cup and ½ more

<u>Instructional Strategies</u>

In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fraction and justify with a fraction model. For example, 3/4 = 1/4 + 1/4 + 1/4.



Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

<u>Common Misconceptions</u>: (4.NB.3-4)

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

Domain: **Number and Operations—Fractions (NF)**

Cluster: **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

Standard: **4.NF.4.** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent* $5/4$ *as the product* $5\times(1/4)$*, recording the conclusion by the equation* $5/4 = 5\times(1/4)$.

b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3\times(2/5)$ as $6\times(1/5)$*, recognizing this product as* $6/5$*.* (*In general,* $n\times(a/b)=(n\times a)/b$*.*)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $3/8$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Standards for Mathematical Practice (MP) to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

Connections:

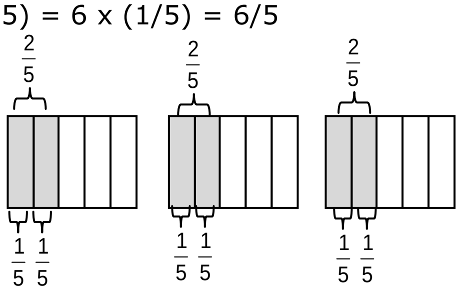
See 4.NF.3

Explanations and Examples:

Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. This standard builds on students' work of adding fractions and extending that work into multiplication. (4.NF.4a)

Examples:

- 3 x (2/5) = 6 x (1/5) = 6/5



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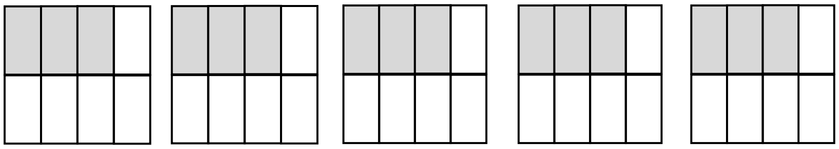
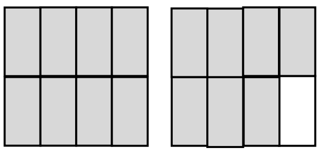
- If each person at a party eats 3/8 of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:



3/8 3/8 3/8 3/8 3/8

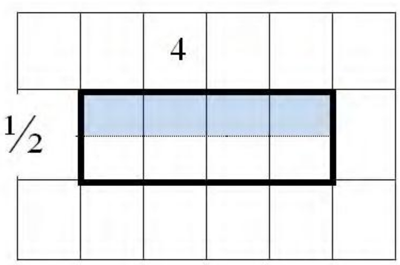


3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1 7/8

This standard extends the idea of multiplication as repeated addition (4.NF.4b) For example,
3 x (2/5) = 2/5 + 2/5 + 2/5= 6/5 = 6 X (1/5). Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

This standard calls for students to use visual fraction models (Area, Linear and Set Models) to solve word problems related to multiplying a whole number by a fraction. (4.NF.4c)

Student 1
Draws a number line to show 4 jumps of ½.
Student 2
Draws and area model showing 4 pieces of ½ joined together to equal 2.
Student 3
Draws an area model representing 4 X ½ on a grid, dividing each row into ½ to represent the multiplier.



Domain: **Number and Operations—Fractions (NF)**

Cluster: **Understand decimal notation for fractions, and compare decimal fractions.**
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =**

Standard: **4.NF.5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. (*Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

<u>Standards for Mathematical Practice (MP) to be emphasized:</u>

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

<u>Connections</u>: **4.NF.5-7**

This cluster is connected to the Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Connect with understanding and generating equivalent fractions (Grade 4 NF 1 – 2).

Students will perform operations with decimals to hundredths in Grade 5 (Grade 5 NBT 5 -7).

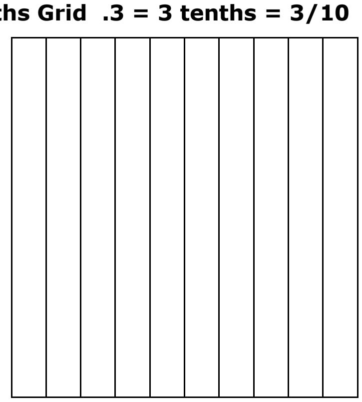
<u>Explanations and Examples:</u>

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. **Student experiences should focus on working with grids rather than algorithms.** Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

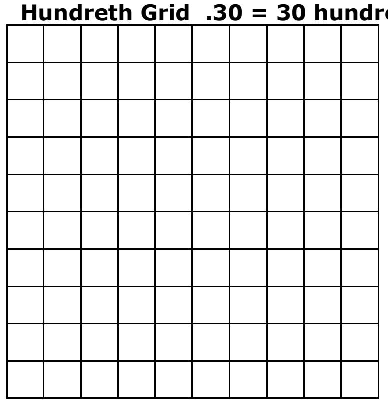
This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.

Ones	.	Tenths	Hundredths

Tenths Grid .3 = 3 tenths = 3/10



Hundreth Grid .30 = 30 hundreths = 30/100



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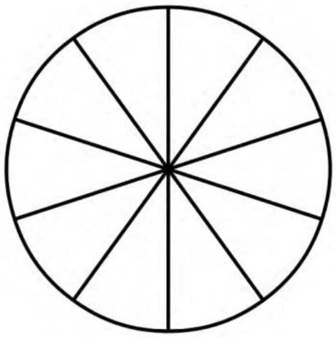
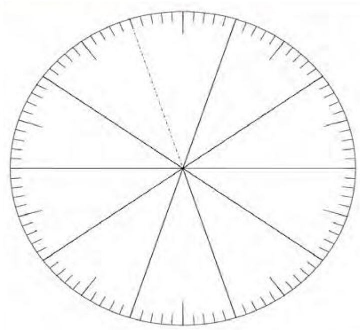
Example:
Represent 3 tenths and 30 hundreths on the models shown below:

10ths circle **100ths circles**

Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

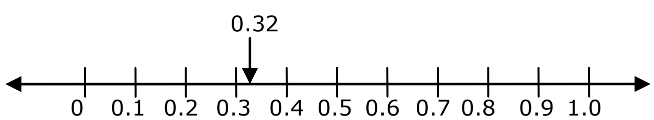
Base Ten Blocks: students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	.	Tenths	Hundreths
			.	3	2

Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100.

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.



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Instructional Strategies

The place value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system. Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

As mentioned above, students can use base-ten blocks to represent decimals. A 10 x 10 block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a 10 x 10 grid.

It is important that students make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100. Have students say the fraction with denominators of 10 and 100 aloud. For example would "four tenths" or 27/100 would be "twenty-seven hundredths." Also, have students represent decimals in word form with digits and the decimal place value, such as 4/10 would be 4 tenths.

Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions (3/10 + 2/100)

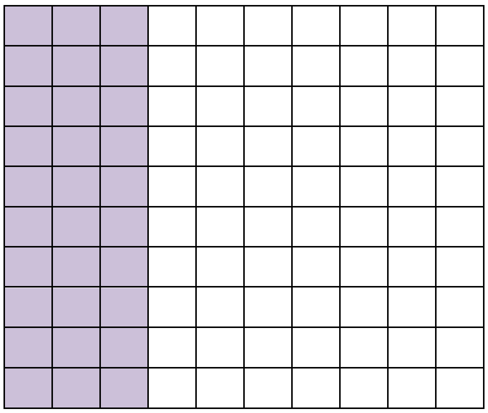
Students' understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system. Understanding of the decimal place value system is important **prior** to the generalization of moving the decimal point when performing operations involving decimals.

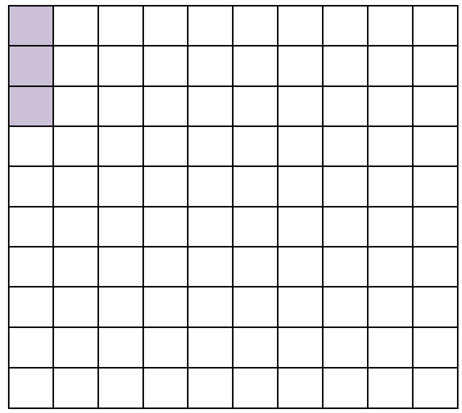
Students extend fraction equivalence from Grade 3 with denominators of 2, 3 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

Continued next page

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.



0.3



0.03

Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Instructional Resources/Tools

Length or area models
10 x 10 square on a grid
Decimal place-value mats
Base-ten blocks
Number lines
Nctm.org (Illuminations): *A Meter of Candy* – In this series of three hands-on activities, students develop and reinforce their understanding of hundredths as fractions, decimals and percentages. Students explore with candy pieces as they physically make and connect a set and linear model (meter) to produce area models (grids and pie graphs). At this time, students are not to do percents. The relationships among fractions, decimals and percents are developed in Grade 6.

Common Misconceptions: 4.NF.5-7

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that).03 is greater than 0.3.

Domain: **Number and Operations—Fractions (NF)**

Cluster: **Understand decimal notation for fractions, and compare decimal fractions.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =**

Standard: **4.NF.6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:

See 4.NF.5

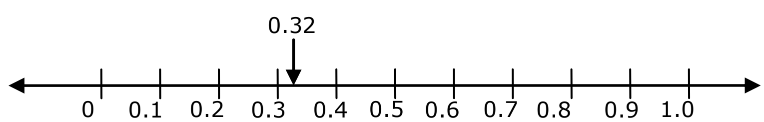
Explanations and Examples:

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100.

Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10). It is closer to 30/100 so it would be placed on the number line near that value.



Domain: **Number and Operations—Fractions (NF)**

Cluster: **Understand decimal notation for fractions, and compare decimal fractions.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =**

Standard: **4.NF.7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Standards for Mathematical Practice (MP) to be emphasized:

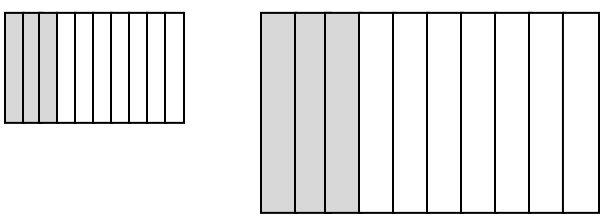
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.7. Look for and make use of structure.

Connections:

See 4.NF.5

Explanations and Examples:

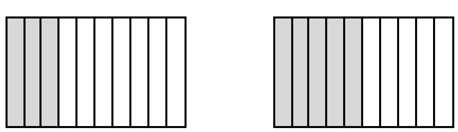
Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.



When the wholes are the same, the decimals or fractions can be compared.

Example:

- Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.



Extended Common Core State Standards
4th Grade

Fourth Grade Mathematics Number and Operations – Fractions (4.NF) North Carolina DOE				
Common Core State Standards		**Essence**	**Extended Common Core**	
Extend understanding of fraction equivalence and ordering		**Extend understanding of fractions**	**Develop understanding of fractions as numbers**	
Cluster	1. Explain why a fraction *a*/*b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.		**Cluster**	1. Identify whole, half, and fourth using concrete models (use continuous and discrete items). 2. Use symbolic representation for each fractional part. 3. Use a number line to identify the half between each number.

Domain: **Measurement and Data (MD)**

Cluster: **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter**

Standard: **4.MD.1.** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs* (*1, 12*), (*2, 24*), (*3, 36*),

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision

Connections: **4.MD.1-3**

This cluster is connected to the Fourth Grade Critical Areas of Focus #1 , **Developing understanding and fluency with multi-digit multiplication , and developing understanding of dividing to find quotients involving multi-digit dividends**, and #2, **Developing an understanding of fractions equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers**.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (Grade 3 MD 8).

Geometric measurement; understand concepts of area and relate area to multiplication and to addition. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Grade 4 NF 3 – 4).

Explanations and Examples:

The units of measure that have not been addressed in prior years are pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass, liquid volume, and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure.

Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.

Examples next page

Example:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (**partition**).
- Understand that the same unit can be repeated to determine the measure (**iteration**).
- Understand the relationship between the size of a unit and the number of units needed (**compensatory principal**).

Instructional Strategies 4.MD.1-3

In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers and yardsticks to discover these relationships among these units of measurements. Using 12-inch rulers and yardstick, students can see that three of the 12-inch rulers, which is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick. Have students record the relationships in a two column table or t-charts. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.

Present word problems as a source of students' understanding of the relationships among inches, feet and yards.

Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12 and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.
Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

Instructional Resources/Tools

Yardsticks(meter sticks) and rulers (marked with customary and metric units)
Teaspoons and tablespoons
Graduated measuring cups (marked with customary and metric units)

Common Misconceptions: 4.MD.1-3

Students believe that larger units will give the larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yard sticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles and explain their reasoning.

Domain: **Measurement and Data (MD)**

Cluster: **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter**

Standard: **4.MD.2.** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Standards for Mathematical Practice (MP) to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

Connections:

See 4.MD.1

Explanations and Examples:

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents).

Students should have ample opportunities to use number line diagrams to solve word problems.

Example:

Debbie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?

Possible solution: Debbie plus 10 friends = 11 total people

11 people x 8 ounces (glass of milk) = 88 total ounces

1 quart = 2 pints = 4 cups = 32 ounces

Therefore 1 quart = 2 pints = 4 cups = 32 ounces
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces

If Debbie purchased 3 quarts (6 pints) of milk there would be enough for everyone at her party to have at least one glass of milk. If each person drank 1 glass then she would have 1- 8 oz glass or 1 cup of milk left over.

Additional examples next page

Additional Examples with various operations:

Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?
Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches.
Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)

Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

Example:
At 7:00 a.m. Melisa wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

Domain: **Measurement and Data (MD)**

Cluster: **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter**

Standard: **4.MD.3.** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*

Standards for Mathematical Practice (MP) to be emphasized:

MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7.Look for and make use of structure.

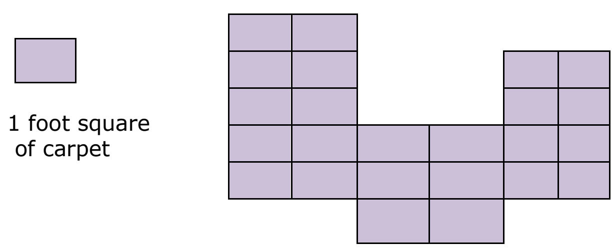
Connections:
See 4.MD.1

Explanations and Examples:

Students developed understanding of area and perimeter in 3rd grade by using **visual** models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to **understand and be able to communicate their understanding of why the formulas work**. The formula for area is *I* x *w* and the answer will always be in square units. The formula for perimeter can be 2 *l* + 2 *w* or 2 (*l* + *w*) and the answer will be in linear units.

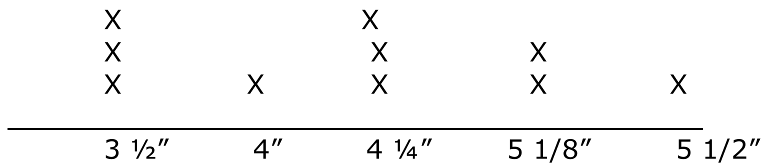
This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:
Mrs. Fields is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will she need to cover the entire course?



Domain: **Measurement and Data (MD)**
Cluster: **Represent and interpret data.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, line plot, length, fractions**
Standard: **4.MD.4**. Make a line plot to display a data set of measurements in fractions of a unit ($^1/_2$, $^1/_4$, $^1/_8$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Standards for Mathematical Practice (MP) to be emphasized: MP.2. Reason abstractly and quantitatively. MP.4. Model with mathematics. MP.5. Use appropriate tools strategically. MP.6. Attend to precision. MP.7. Look for and make use of structure.
Connections: This cluster is connected to the Fourth Grade Critical Area of Focus #2 , **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** Understand a fraction as a number on the number line; represent fractions on a number line diagram (Grade 3 NF 2). Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size (Grade 3 NF 3). Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters (Grade 3 MD 4). Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem (Grade 4 NF 3d).
Explanations and Examples: This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot. **Example**: Students measured objects in their desk to the nearest ½, ¼, or 1/8 inch. They displayed their data collected on a line plot. How many object measured ¼ inch? ½ inch? If you put all the objects together end to end what would be the total length of **all** the objects. Examples continued next page

Ten students in Room 31 measured their pencils at the end of the day. They recorded their results on the line plot below.



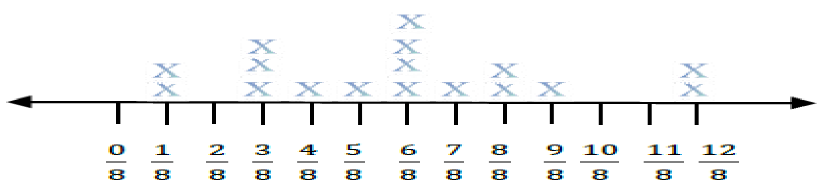
Possible questions:

- What is the difference in length from the longest to the shortest pencil?
- If you were to line up all the pencils, what would the total length be?
- If the 5 1/8″ pencils are placed end to end, what would be their total length?

Instructional Strategies

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit (1/2, 1/4 , 1/8) and plot data showing multiple data points for each fraction.



Pose questions that students may answer, such as:

- “How many one-eighths are shown on the line plot?” Expect “two one-eighths” as the answer. Then ask, “What is the total of these two one-eighths?” Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- “What is the total number of inches for insects measuring 3/8 inches?” Students can use skip counting with fraction names to find the total, such as, “three-eighths, six-eighths, nine-eighths. The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. have them make a statement about the result of adding fractions with the same denominator.
- “What is the total number of insects measuring 1/8 inch or 5/8 inches?” Have students write number sentences to represent the problem and solution such as,
  1/8 + 1/8 + 5/8 = 7/8 inches.

Use visual fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Continued next page

Instructional Resources/Tools
Fraction bars or strips Number Lines
Common Misconceptions: Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths. Students also count the tick marks on the number line to determine the fraction, rather than looking at the "distance" or "space" between the marks.

Domain: **Measurement and Data (MD)**

Cluster: **Geometric measurement: understand concepts of angle and measure angles.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown**

Standard: **4.MD.5.** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $^1/_{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.
b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees

Standards for Mathematical Practice (MP) to be emphasized:

MP.5. Use appropriate tools strategically
MP.6. Attend to precision.
MP.7. Look for and make use of structure.

Connections: **4.MD.5-7**

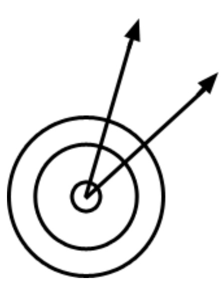
This cluster is connected to the Fourth Grade Critical Area of Focus #3, **Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.**

Connect measuring angle to the Geometry domain in which students draw and identify angles as right, acute and obtuse (Grade 4 G 1).

Explanations and Examples:

This standard calls for students to explore the connection between angles (measure of rotation) and circular measurement (360 degrees).

The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.

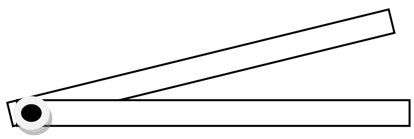


Students explore an angle as a series of "one-degree turns." A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

Instructional Strategies next page

Instructional Strategies
Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.



They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.
Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Instructional Resources/Tools
Cardboard cut in strips to make an angle explorer
Brass fasteners
Protractor
Angle ruler
Straws
Transparencies
Angle explorers

Continued next page

Sir Cumference and the Great Knight of Angleland: In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest, the successful completion of which will earn him his knighthood. With the help of a family heirloom that functions much like a protractor, he is able to locate the elusive King Lell and restore him to the throne of Angleland. In gratitude, King Lell bestows knighthood on Sir Radius.

Nctm.org Figure This: What's My Angle? math Challenge # 10 - Students can estimate the measures of the angles between their fingers when they spread out their hand.

Common Misconceptions: 4.MD 5-7

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should have multiple experiences estimating and comparing angles to the Benchmark 90° or right angle. They should explain their reasoning by deciding first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

Domain: **Measurement and Data (MD)**

Cluster: **Geometric measurement: understand concepts of angle and measure angles.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown**

Standard: **4.MD.6.** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure

<u>Standards for Mathematical Practice (MP) to be emphasized:</u>

MP.2. Reason abstractly and quantitatively.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

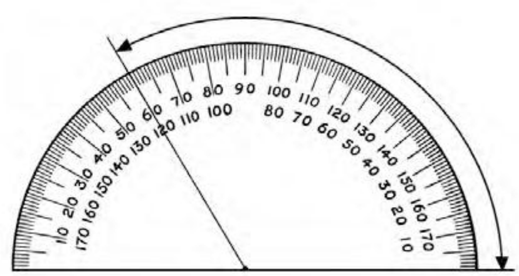
<u>Connections</u>:

See 4.MD.5

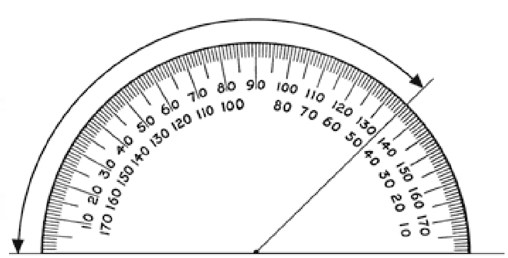
<u>Explanations and Examples:</u>

Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should estimate angles, measure angles and sketch angles. They need to experience measuring angles using an *angle ruler* as well as a *protractor. (The angle ruler allows them to "see" the turns or rotations).*



120 degrees



135 degree

Domain: **Measurement and Data (MD)**

Cluster: **Geometric measurement: understand concepts of angle and measure angles.** Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown**

Standard: **4.MD.7.** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Standards for Mathematical Practice (MP) to be emphasized:

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.4. Model with mathematics.
MP.6. Attend to precision.

Connections:

See 4.MD 5

Explanations and Examples:

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.

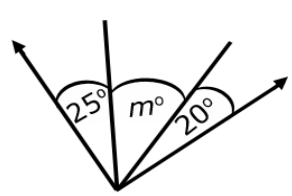
Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of m?

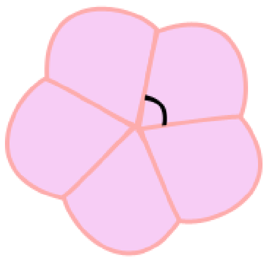


Examples continued next page

Example:

- Joe Dan knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

- The five shapes in the diagram are the exact same size. Write an equation that will help you find the measure of the indicated angle. Find the angle measurement.



Extended Common Core State Standards
Mathematics
North Carolina DOE

<table>
<tr><th colspan="5">Fourth Grade Mathematics
Measurement and Data (4.MD)</th></tr>
<tr><th colspan="2">Common Core State Standards</th><th>Essence</th><th colspan="2">Extended Common Core</th></tr>
<tr><td colspan="2">Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit</td><td rowspan="2">Solve problems involving measurement</td><td colspan="2">Solve problems involving measurement time and mass</td></tr>
<tr><td>Cluster</td><td>1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24),
(3, 36), ...
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement
scale.
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</td><td>Cluster</td><td>1. Tell time to the nearest hour.
2. Compare two objects using direct comparison of mass.
3. Solve problems using appropriate vocabulary to describe differences in weight (e.g. more, less, same).
4. Use customary unit to measure weight (ounces and pounds).</td></tr>
<tr><td rowspan="2">Cluster</td><td>Represent and interpret data</td><td rowspan="2">Represent and interpret data</td><td rowspan="2">Cluster</td><td>Represent and interpret data</td></tr>
<tr><td>4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</td><td>4. Organize and represent data using bar graphs.
5. Title and label axis of graph.
6. Answer questions posed about the collected data.</td></tr>
</table>

EXTENDED 4.MD.

Domain: **Geometry (G)**

Cluster: **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**
Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Standard: **4.G.1**. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

<u>Standards for Mathematical Practice (MP) to be emphasized:</u>
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.

<u>Connections</u>: **4.G.1-3**
This cluster is connected to the Fourth Grade Critical Area of Focus #3**, Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.**

Geometric measurement: understand concepts of angles and measure angles (Grade 4 MD 3).

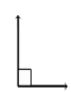
Symmetry can be related to experiences in art.

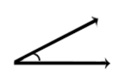
<u>Explanations and Examples:</u>
This standard asks students to draw two-dimensional geometric objects and to also identify them in two- dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines.

Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily.

Students do not easily identify lines and rays because they are more abstract.

Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract.


Right angle


Acute angle


Obtuse angle


Straight angle

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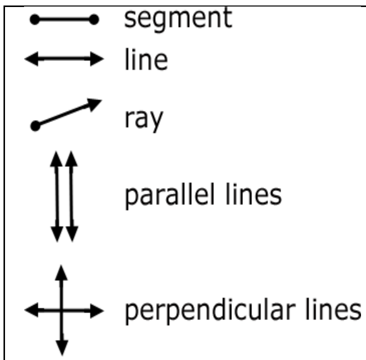
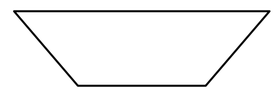


Example:
Draw two different types of quadrilaterals that have two pairs of parallel sides?
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.

Example:
How many acute, obtuse and right angles are in this shape?



Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

Instructional Strategies

Angles

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Symmetry

When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.
With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Continued next page

Two-dimensional shapes
Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Informal understanding of the characteristics of triangles is developed through angle measures and side length relationships. Triangles are named according to their angle measures (right, acute or obtuse) and side lengths (scalene, isosceles or equilateral). These characteristics are used to draw triangles.

Instructional Resources/Tools
Mirrors
Geoboards
GeoGebra (a free software for learning and teaching); http://www.geogebra.com.

Common Misconceptions:
Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

Domain: **Geometry (G)**

Cluster: **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Standard: **4.G.2.** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Standards for Mathematical Practice (MP) to be emphasized:

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

Connections:

See 4.G.1

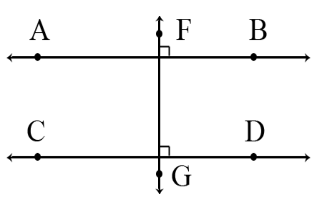
Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:

Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º).

Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:



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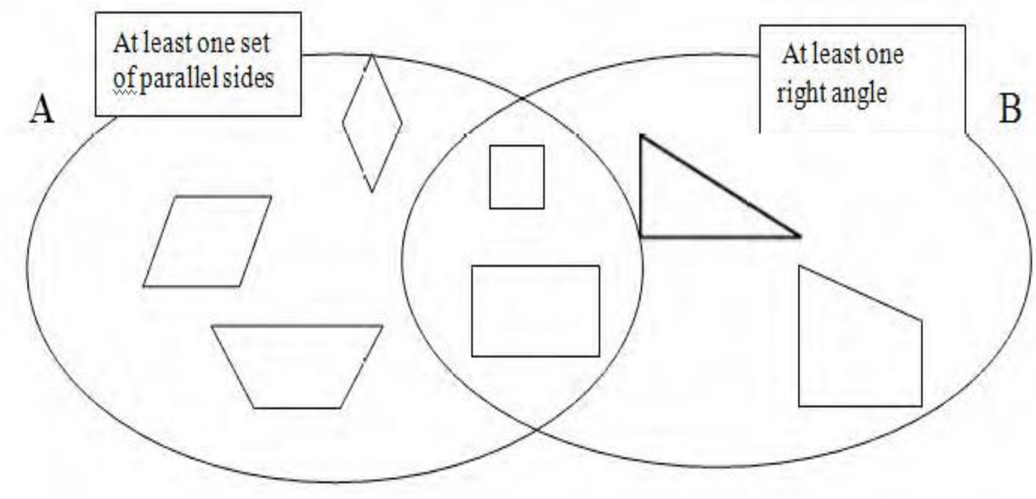
This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.

Example:



Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

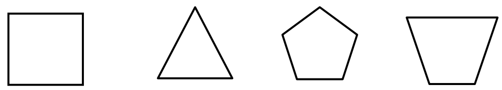
Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show counter example.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

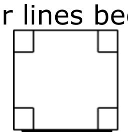
Examples continued next page

Example:

- Identify which of these shapes have perpendicular or parallel sides and justify your selection.



A possible justification that students might give is:
The square has perpendicular lines because the sides meet at a corner, forming right angles.



Angle Measurement:

This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

Domain: **Geometry (G)**

Cluster: **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Standard: **4.G.3.** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Standards for Mathematical Practice (MP) to be emphasized:

MP.4. Model with mathematics.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7.Look for and make use of structure.

Connections:

See 4.G.1

Explanations and Examples:

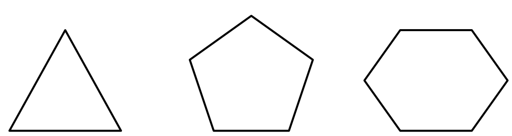
Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

This standard only includes line symmetry not rotational symmetry.

Example:

For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.

Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.



Extended Common Core State Standards (NC DOE)

Fourth Grade Mathematics Geometry (4.G)				
Common Core State Standards		Essence	Extended Common Core	
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.			**Identify lines, angles, and properties of a shape (circle, square, rectangle, triangle, and rhombus).**	
Cluster	1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.		Cluster	1. Identify angles in each shape. 2. Describe the attributes of two-dimensional shapes (i.e., number sides and angles, straight vs curved lines).

Table 1. Common addition and subtraction situations.[34]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[35]
Put Together/ Take Apart[36]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[37]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[34] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[35] These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[36] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[37] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

TABLE 2. Common multiplication and division situations.[38]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 × 6 = ?	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[39] Area[40]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[38] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[39] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[40] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.